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APPLICATION OF MULTIPLE SCATTERING THEORY TO SUB-SURFACE DEFECTS

E. Domany* and O. Entin-Wohlman**

*Department of Electronics, Weizmann Institute of Science, Rehovot, Israel
**Department of Physics and Astronomy, Tel Aviv University Ramat Aviv, Israel

ABSTRACT

A recently developed multiple scattering formalism is applied to treat scattering of elastic waves by subsurface defects. In particular, the problem of a spherical cavity near a stress free surface is treated.

INTRODUCTION

There has been considerable progress in development of various approximation methods to scattering of elastic waves by bulk defects.[1] Complex scatterer geometries were treated[2,3] and results of various approximations were compared to each other and to experiment.[4] Some simple approximation methods were used with considerable success to generate inversion schemes.[5] Thus it is a useful and natural extension of past work to turn to treat defects which are of NDE interest and involve more complex methods of analysis, such as subsurface cavities, cracks and inclusions. In the present study we report calculations of scattering of elastic waves by a spherical cavity in the vicinity of a stress-free planar surface. We use a recently developed multiple scattering formalism,[6] as well as the results of previous calculations on scattering by two adjacent cavities.[3] In the present study the spherical cavity is viewed as one scatterer, and the stress free plane (reflector) as the second. The solution to the sub-surface cavity is then represented as an expansion in the two scattering processes. This expansion is truncated after a finite number of terms; namely, only processes that include single scattering by the sphere are kept.

The neglected terms involve at least double scattering by the sphere, which process can be interpreted as scattering by the sphere and consecutively by its image. Thus this is identical to the multiple scattering process that was studied previously in our work on scattering by two spherical cavities,[3] and which was shown to have a very small contribution even for separations comparable to the spheres' diameter.

We first review briefly the multiple scattering formalism, and present physical interpretation of the various terms in the expansion. Then the contribution of each such term is expressed in terms of scattering amplitudes of a spherical cavity in an infinite medium and the reflection coefficients of a stress-free planar surface. Our results are summarized in terms of graphs of scattered amplitude as a function of frequency and scattering angle.

MULTIPLE SCATTERING FORMALISM: A BRIEF REVIEW

Since the general multiple scattering formalism on which this work is based was presented in detail elsewhere,[6] here only a brief summary of the main results is given. We assume that two defects, "1" and "2", are embedded in an infinite elastic medium, characterized by Lamé coefficients λ, μ, and density ρ; ω is the frequency of the incident (longitudinal) wave u^o and $\alpha(\beta)$ are the longitudinal (shear) wave vectors.

The solution of the scattering problem, with only one scatterer j present is given by ($j = 1,2$)

$$u^{(j)} = u^o + u^{(j)S} \tag{1}$$

where

$$u^{(j)S} = g^o T^{(j)} u^o \tag{2}$$

here g^o is the infinite medium Green's function and $T^{(j)}$ is the T-matrix associated with scatterer j.

Similarly, the Green's function of the problem with only scatterer j present can be written as

$$g^{(j)} = g^o + g^{(j)S} , \tag{3}$$

$$g^{(j)S} = g^o \, T^{(j)} \, g^o . \tag{4}$$

The solution of the scattering problem with both scatterers $j = 1,2$ present, u, can be expressed as

$$u = u^o + u^S \tag{5}$$

$$u^S = g^o \; T \; u^o . \tag{6}$$

T, the T-matrix of the composite scatterer (1,2 acting together) can be expanded in terms of the $T^{(j)}$;

$$T = T^{(1)} + T^{(2)} + T^{(1)} g^o \; T^{(2)} + \\ + T^{(2)} \; g^o \; T^{(1)} + T^{(2)} \; g^o \; T^{(1)} \; g^o \; T^{(2)} + \ldots \tag{7}$$

Substituting this expansion in (6), and neglecting all terms not explicitly present in (7), one obtains for the scattered wave the approximate expansion

$$u^S \cong g^o \; T^{(1)} \; u^o + g^o \; T^{(2)} \; u^o + \\ + g^o \; T^{(1)} \; g^o \; T^{(2)} \; u^o \quad + \quad g^o \; T^{(2)} \; g^o \; T^{(1)} \; u^o \\ + g^o \; T^{(2)} \; g^o \; T^{(1)} \; g^o \; T^{(2)} \; u^o \tag{8}$$

We now consider these terms one by one. Recapitulating (2) we get

$$g^o \; T^{(j)} \; u^o = u^{(j)S} \; , \tag{9}$$

and

$$g^o \; T^{(1)} \; g^o \; T^{(2)} \; u^o = g^o \; T^{(1)} \; u^{(2)S} \tag{10}$$

while (4) yields

$$g^o \; T^{(2)} \; g^o \; T^{(1)} \; u^o = g^{(2)S} \; T^{(1)} \; u^o \tag{11}$$

Finally, using both (2) and (4), the relation

$$g^o \; T^{(2)} \; g^o \; T^{(1)} \; g^o \; T^{(2)} \; u^o = g^{(2)S} \; T^{(1)} \; u^{(2)S} \tag{12}$$

is found.

Note that in the problem at hand "2" denotes the stress-free planar reflector; as we will demonstrate, both $g^{(2)S}$ and $u^{(2)S}$ can be expressed in terms of a finite number of plane waves; and therefore in the far-field limit all the contributions of interest can be expressed in terms of the scattering amplitude of a single spherical cavity in an infinite medium. It should be noted that these simple formal manipulations eliminate the need for complex (and expensive) $\underline{k}$-space integrations.

APPLICATION TO SCATTERING BY SPHERICAL CAVITY NEAR A STRESS-FREE PLANE

Before embarking on the explicit evaluation of the formal expressions (9)-(12), we pause to give a physical interpretation to each of these terms.

We consider the scattering geometry of Fig. 1. The first two terms (eq. (9)) represent direct scattering processes by the sphere and by the plane. The contribution of these to the total $L \to L$ amplitude was sketched in Fig. 2. In what follows, the directly reflected wave will not be included; it contributes only when the condition of specular reflection is satisfied by the directions of incidence ($\hat{\alpha}_o$) and observation ($\hat{r}$).

The term of eq. (10) represents the process indicated in Fig. 3, i.e. reflection by the plane followed by scattering by the sphere. Note that in Fig. 3b the incident wave is mode converted by the plane into a shear wave, and subsequently converted again into an L wave by the sphere. The term of equation (11) represents processes of scattering by the sphere, followed by reflection to the direction of observation by the plane, as shown in Fig. 4. Finally, eq. (12) contains the processes indicated in Fig. 5, of which all but the last are explicitly calculated below. The last process involves shear-to-shear scattering by a spherical cavity, and will be evaluated elsewhere.

In order to explicitly calculate the formal expressions (9)-(12), we have to recapitulate some known results concerning the scattering problems "1" and "2", i.e. scattering by a spherical cavity and reflection by a stress-free plane.

For a longitudinal plane wave in the +z direction ($\underline{k}_o = \alpha\hat{z}$)

$$\underline{u}^o(\underline{k}_o;\underline{r}) = \hat{k}_o\, e^{i\underline{k}_o\cdot\underline{r}}$$

incident on a spherical cavity with its center at $\underline{r} = 0$, the scattered wave at $\underline{r} = \hat{r}\cdot r$, $r \to \infty$ has the form

$$u^{(1)S}(\underline{k}_o) = \hat{r}\, A\, \frac{e^{i\alpha r}}{r} + \hat{\theta}\, B\, \frac{e^{i\beta r}}{r} \tag{13}$$

where the amplitudes A and B are functions of the scattering angle (between $\underline{k}_o$ and $\underline{k} = \alpha\hat{r}$) and the frequency (or α). These amplitudes are readily obtained from the exact solution of the problem.[7] We now turn to relate this form of $u^{(1)S}$ to that of equation (2), which stands for the full expression

$$u_i^{(1)S}(r) = \sum_{j,\ell} \int d\underline{r}'\, g_{ij}^{\,o}\,(\underline{r},\underline{r}')\, T_{j\ell}^{(1)}\, u_\ell^{\,o}(\underline{k}_o;\underline{r}') \tag{14}$$

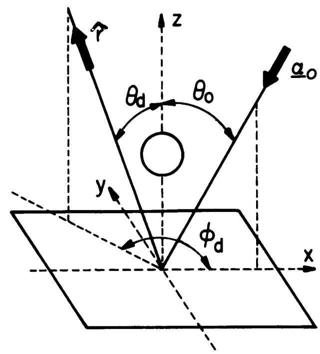

Fig. 1 Scattering geometry: sphere of radius a centered at (o,o,d,); direction of incidence $\underline{\alpha}_o$ in the x-z plane, at angle θ_o with normal to plane. Detector at $\hat{r}$, defined by polar angle θ_d and azymuthal angle ϕ_d.

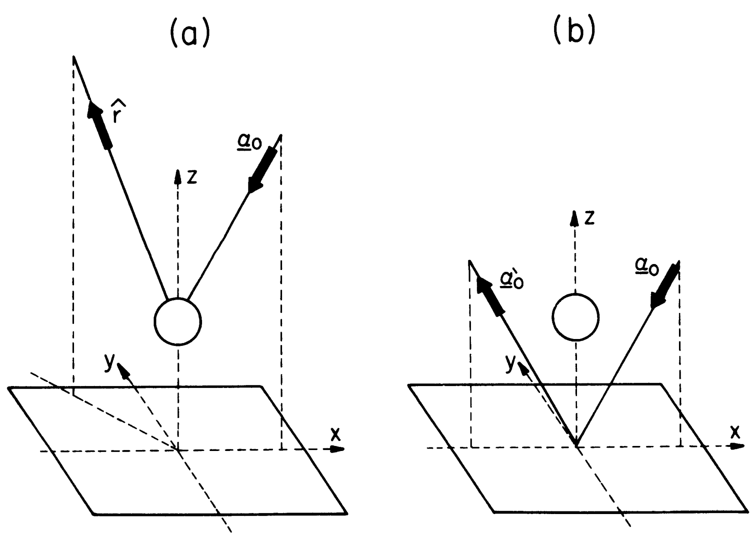

Fig. 2 Direct scattering processes: (a) by the sphere, and (b) by the plane. (b) contributes only in the direction of specular reflection $\underline{\alpha}_o'$.

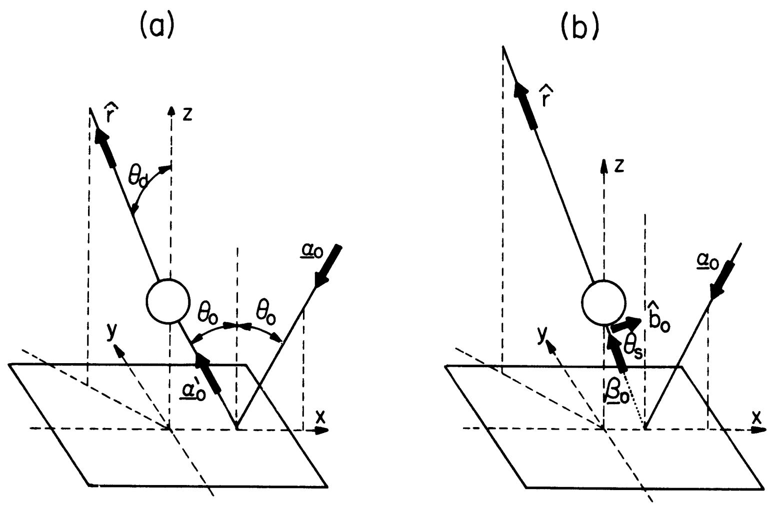

Fig. 3 The processes of equation (10); (a) Longitudinal reflected wave scattered by the sphere: (b) Mode-converted reflected shear wave, with wave vector $\underline{\beta}_o$ and polarization $\hat{b}_o$, scattered (and mode-converted) by the sphere.

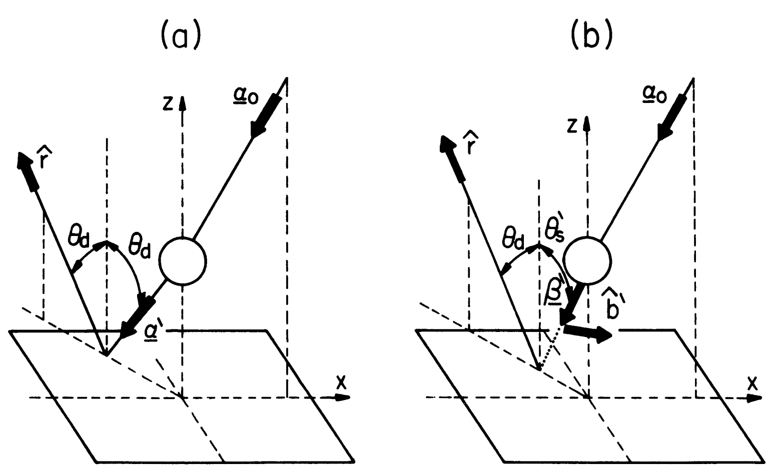

Fig. 4 Scattering by the sphere creates (a) Longitudinal and (b) Shear waves, subsequently reflected by the plane. The process corresponds to equ. (11).

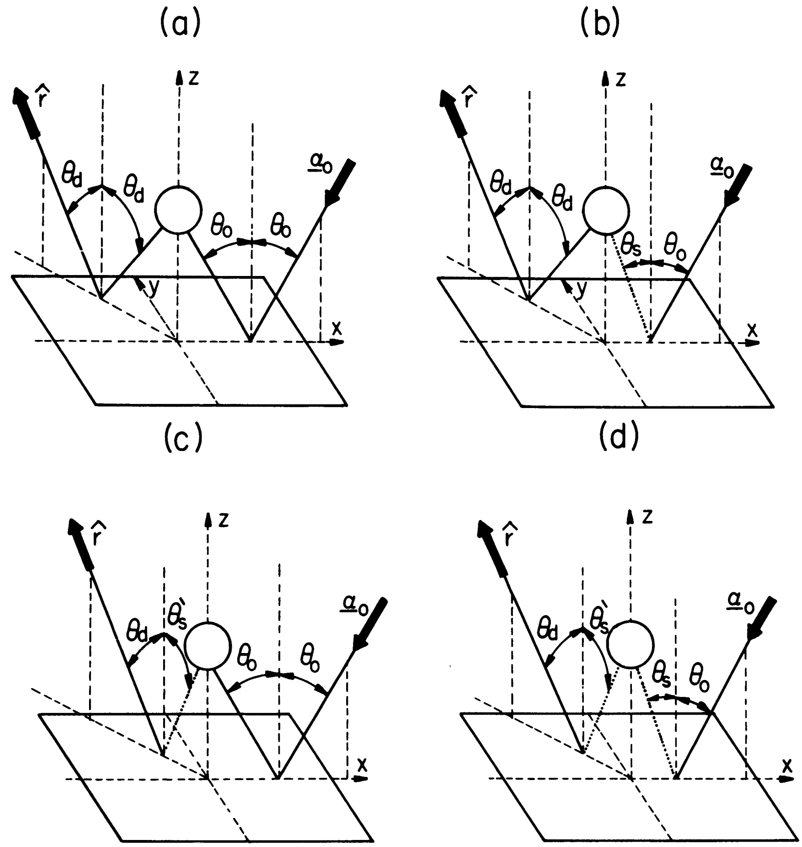

Fig. 5 Processes of equ. (12); reflection by plane before and after scattering by sphere.

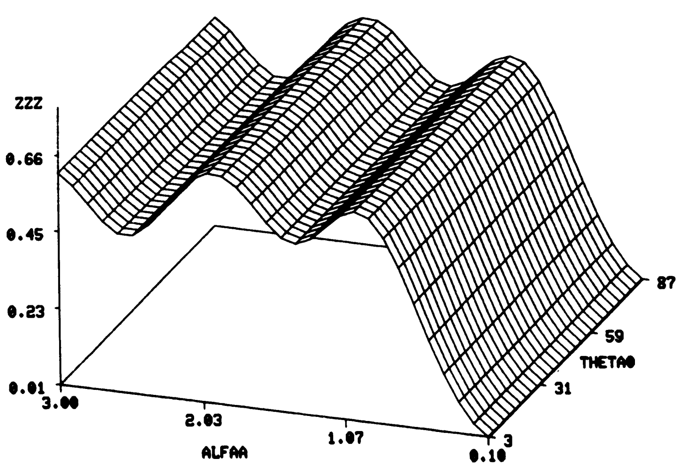

Fig. 6 Direct L-L scattering amplitude $A(\theta)$ vs. αa and θ_o.

in the $r \to \infty$ limit the expansion

$$g^o_{ij}(\underline{r},\underline{r}') = \frac{1}{4\pi\rho\omega^2} \sum_\varepsilon \hat{e}^\varepsilon_i \frac{e^{i\gamma(\varepsilon)r}}{r} \gamma(\varepsilon)^2 \hat{e}^\varepsilon_j e^{-i\gamma(\varepsilon)\hat{r}\cdot\underline{r}'} \qquad (15)$$

is now used, where ε denotes various polarizations; $\varepsilon = 1$ stands for longitudinal $[\hat{e}^1 = \hat{r},\ \gamma(1) = \alpha]$ while $\varepsilon = 2,3$ for shear waves $[\hat{e}^2, \hat{e}^3 = \hat{\theta}, \hat{\phi};\ \gamma(2,3) = \beta]$. Substituting (15) into (14) we get precisely the form (13), with the identification $[\cos\theta = \hat{r}\cdot\hat{k}^o]$

$$\begin{aligned} A(\theta) &= \frac{\alpha^2}{4\pi\rho\omega^2} \int d\underline{r}'\, \hat{r}_j\, e^{-i\alpha\hat{r}\cdot\underline{r}'}\, T^{(1)}_{j\ell}\, \hat{k}^o_\ell\, e^{i\alpha\hat{k}^o\cdot\underline{r}'} \\ &\equiv \frac{\alpha^2}{4\pi\rho\omega^2} < u^o(\alpha\hat{r})\,|T^{(1)}|\,u^o(\alpha\hat{k}^o) > \end{aligned} \qquad (16)$$

that relates the scattering amplitude A (Longitudinal to Longitudinal) at angle θ to the appropriate "element" of the T-matrix. For the mode converted (Longitudinal to Shear) wave we get

$$\begin{aligned} B(\theta) &= \frac{\beta^2}{4\pi\rho\omega^2} \int d\underline{r}'\, \hat{\theta}_i\, e^{-i\beta\hat{r}\cdot\underline{r}'}\, T^{(1)}_{ij}\, \hat{k}^o_j\, e^{i\alpha\hat{k}^o\cdot\underline{r}'} \\ &\equiv \frac{\beta^2}{4\pi\rho\omega^2} < u^o(\beta\hat{r})\,|T^{(1)}|\,u^o(\alpha\hat{\underline{k}}^o) > \end{aligned} \qquad (17)$$

An additional result[8] that will be utilized below relates the amplitude of the (mode converted) scattered longitudinal wave with wave vector $\underline{\alpha}$, for an incident shear wave with wave vector $\underline{\beta}$, and polarization $\hat{b}$, e.g. $A_{\hat{b}}(\underline{\beta},\underline{\alpha})$, to that of a (mode converted) scattered shear wave, with polarization $\hat{b}'$, and wave vector $-\underline{\beta}$, for an incident longitudinal wave with wave vector $-\underline{\alpha}$;

$$A_{\hat{b}}(\underline{\beta},\underline{\alpha}) = -\frac{\alpha^2}{\beta^2}(\hat{b}\cdot\hat{b}')\, B(-\underline{\alpha}, -\underline{\beta}) \qquad (18)$$

We now turn to reflection by a stress-free plane; for an incident longitudinal wave with wave vector $\underline{\alpha}_o$ (at angle θ_o with the normal to the plane), the reflected longitudinal and shear waves have wave vectors and polarization as indicated in Fig. 2b, and respective amplitudes[9]

$$\delta_L(\theta_o) = \frac{\alpha^2\sin 2\theta_o \sin 2\theta_s - \beta^2\cos^2 2\theta_s}{\Delta} \qquad (19)$$

$$\delta_S(\theta_o) = -\frac{2\alpha\beta\sin 2\theta_o \cos 2\theta_s}{\Delta} \qquad (20)$$

$$\Delta = \alpha^2\sin 2\theta_s \sin 2\theta_o + \beta^2\cos^2 2\theta_s$$

where θ_s is defined by

$$\alpha\sin\theta_o = \beta\sin\theta_s$$

Thus

$$\underline{u}^{(2)S}(\underline{\alpha}_o) = \delta_L \hat{\alpha}_o' e^{i\underline{\alpha}_o'\cdot\underline{r}} + \delta_s \hat{b}_o e^{i\underline{\beta}_o\cdot\underline{r}} \tag{21}$$

Here we use $\underline{\beta}_o = \beta\hat{\beta}_o$, $\alpha\sin\theta_o = \beta\sin\theta_s$, and for incidence along

$$\hat{\alpha}_o = -\hat{z}\cos\theta_o - \hat{x}\sin\theta_o$$

we have

$$\hat{\beta}_o = \hat{z}\cos\theta_s - \hat{x}\sin\theta_s$$

$$\hat{\alpha}_o' = \hat{z}\cos\theta_o - \hat{x}\sin\theta_o$$

$$\hat{b}_o = \hat{z}\sin\theta_s + \hat{x}\cos\theta_s \tag{22}$$

The last quantity we need is $g^{(2)S}$, the scattered part of the Green's function associated with a planar reflector. This function is given by ($\underline{r}$ denotes the point of observation, r' the source)

$$g_{ij}^{(2)S}(\underline{r},\underline{r}') = \frac{-1}{4\pi\rho_o\omega^2}\sum_{\varepsilon} \hat{e}_i^{\varepsilon} \frac{e^{i\gamma(\varepsilon)r}}{r}\gamma(\varepsilon)^2\cdot u_j^{(2)S}[\underline{r}',-\gamma(\varepsilon)\hat{r},\varepsilon] \tag{23}$$

Here $u_j^{(2)S}[\underline{r}',-\gamma(\varepsilon)\hat{r},\varepsilon]$ stands for the wave scattered by the reflector plane, corresponding to incident wave vector $-\gamma(\varepsilon)\hat{r}$ and polarization ε, evaluated at $\underline{r}'$. For example, the longitudinal ($\varepsilon = L$, $\hat{e}^{\varepsilon} = \hat{r}$, $\gamma(\varepsilon) = \alpha$) part contains the scattered wave (evaluated using (21)), that corresponds to an incident plane wave

$$\underline{u}^o = (-\hat{r})e^{-i\alpha\hat{r}\cdot\underline{r}'}$$

A final note is needed, to account for phase factors. We quoted solutions to the planar reflector, appropriate for a coordinate system with its origin on the plane, and for the spherical scatterer we use a coordinate system with origin at the sphere's center. Therefore in (21) and (23) the substitution

$$\underline{r} \to \underline{r} + \underline{d} \qquad ; \qquad \underline{r}' \to \underline{r}' + \underline{d}$$

has to be used. This gives rise to phase factors. For incidence on the plane before scattering by the sphere the phase factors are $\exp(i\phi_o^{(L)})$ and $\exp(i\phi_o^{(S)})$, for reflected longitudinal and shear waves, respectively. Similarly, for incidence on the plane after scattering by the sphere, the phase factors are $\exp(i\phi_d^{(L)})$ and $\exp(i\phi_d^{(S)})$. We have (for definition of angles see Fig. 1):

$$\begin{aligned}
\phi_L(\theta_o) &= 2\alpha d\cos\theta_o \\
\phi_S(\theta_o) &= \beta d\cos\theta_s + \alpha d\cos\theta_o \\
\phi_L(\theta_d) &= 2\alpha d\cos\theta_d \\
\phi_S(\theta_d) &= \beta d\cos\theta_s{}' + \alpha d\cos\theta_d
\end{aligned} \tag{24}$$

Obviously, for the processes of Fig. 5, with incidence on the plane before and after scattering by the sphere, both ϕ_o and ϕ_d are needed.

Substituting all the above derived expressions into equations (9)-(12), we obtain for the scattered amplitude

$$A^{Scatt} = A^{dir} + A^{int} + A^{PSP} \tag{25}$$

A^{dir} is the amplitude associated with direct scattering by the sphere;

$$A^{dir} = A(\theta) \quad ; \cos\theta = - [\cos\theta_o \cos\theta_d + \sin\theta_o \sin\theta_d \cos\phi_d]$$

The processes of Fig. 3,4 are given by A^{int};

$$A^{int} = A^{PS} + A^{SP} \tag{26}$$

where A^{PS} represents reflection by the plane followed by scattering by the sphere, and A^{SP} scattering followed by reflection.

Each of these amplitudes is decomposed into an L-L and S-L part, depending on whether the incident longitudinal wave is mode converted (by plane or sphere) or not.

$$A^{PS} = A^{PS}_{LL} + A^{PS}_{SL} \tag{27}$$

where

$$A^{PS}_{LL} = \delta_L(\theta_o)\ e^{i\phi_L(\theta_o)}\ A(\theta_{LL}) \tag{28}$$

$$A^{PS}_{SL} = \delta_S(\theta_o) e^{i\phi_S(\theta_o)} (- \frac{\alpha^2}{\beta^2}) (\hat{b}\cdot\hat{b}')\ B(\theta_{SL}) \tag{29}$$

In the expression for A^{PS}_{SL} we used the reciprocity relation (18); the polarization factor is

$$(\hat{b}\cdot\hat{b}') = (\cos\theta_s \sin\theta_d \cos\phi_d + \sin\theta_s \cos\theta_d)/\sin\theta_{SL}$$

where the scattering angles are defined by

$$\cos\theta_{LL} = \cos\theta_o \cos\theta_d - \sin\theta_o \sin\theta_d \cos\phi_d \tag{30}$$

$$\cos\theta_{SL} = \cos\theta_s \cos\theta_d - \sin\theta_s \sin\theta_d \cos\phi_d \tag{31}$$

Similarly,

$$A^{SP} = A^{SP}_{LL} + A^{SP}_{LS} \tag{32}$$

with

$$A^{SP}_{LL} = \delta_L(\theta_d)\ e^{i\phi_L(\theta_d)} A(\theta_{LL}) \tag{33}$$

$$A^{SP}_{LS} = -\delta_s(\theta_d) e^{i\phi_s(\theta_d)}\ \tilde{b} \cdot \frac{\alpha^2}{\beta^2} B\ (\theta_{LS}) \tag{34}$$

where θ_{LL} is again given by (30), while

$$\cos\theta_{LS} = \cos\theta_o \cos\theta'_s - \sin\theta_o \sin\theta_s' \cos\phi_d \tag{35}$$

and

$$\tilde{b} = (\cos\theta_s' \sin\theta_o \cos\phi_d + \sin\theta_s' \cos\theta_o)/\sin\theta_{LS} \tag{36}$$

Turning now to the final contributions, of reflection by the plane, followed by scattering and reflection again, we calculated three of the four processes of Fig. 5. That is, while

$$A^{PSP} = A^{PSP}_{LL} + A^{PSP}_{LS} + A^{PSP}_{SL} + A^{PSP}_{SS}$$

we have not calculated A^{PSP}_{SS}; this will be done in the future. The terms we do have are given by

$$A^{PSP}_{LL} = \delta_L(\theta_o)\ e^{i\phi_L(\theta_o)} A(\bar{\theta}_{LL})\ e^{i\phi_L(\theta_d)}\ \delta_L(\theta_d) \tag{37}$$

$$A^{PSP}_{LS} = \delta_L(\theta_o)\ e^{i\phi_L(\theta_o)} B(\bar{\theta}_{LS})\ \left(\frac{\alpha^2}{\beta^2}\right)\ \bar{b}\ e^{i\phi_S(\theta_d)}\ [-\delta_s(\theta_d)] \tag{38}$$

$$A^{PSP}_{SL} = \delta_s(\theta_o)\ e^{i\phi_s(\theta_o)} B(\bar{\theta}_{SL})\ \left(-\frac{\alpha^2}{\beta^2}\right)\ \bar{\bar{b}}\ e^{i\phi_L(\theta_d)}\ \delta_L(\theta_d) \tag{39}$$

where

$$\bar{b} = (\sin\theta_o \cos\theta_s' \cos\phi_d - \cos\theta_o \sin\theta_s')/\sin\bar{\theta}_{LS} \tag{40}$$

$$\bar{\bar{b}} = (\sin\theta_d \cos\theta_s \cos\phi_d - \cos\theta_d \sin\theta_s)/\sin\bar{\theta}_{SL} \tag{41}$$

with the scattering angles given by

$$\cos\bar{\theta}_{LL} = -(\sin\theta_o \sin\theta_d \cos\phi_d + \cos\theta_o \cos\theta_d) \tag{42}$$

$$\cos\bar{\theta}_{LS} = -(\sin\theta_o \sin\theta_s' \cos\phi_d + \cos\theta_o \cos\theta_s') \tag{43}$$

$$\cos\bar{\theta}_{SL} = -(\sin\theta_d \sin\theta_s \cos\phi_d + \cos\theta_d \cos\theta_s) \tag{44}$$

RESULTS AND DISCUSSION

Figures 6, 7 and 8 summarized the numerical results obtained. Various scattering amplitudes are plotted, as functions of (frequency) αa, and the angle of incidence, θ_o. The geometry used is one of backscattering from a spherical cavity in Titanium, of radius a, at distances d = 3a and d = 6a from a stress free planar surface.The scattered longitudinal wave (for longitudinal incidence) is presented.

Fig. 6 displays the longitudinal amplitude directly backscattered by the sphere (in units of a). This amplitude is obviously independent of θ_o and similar for d/a = 3 and 6.

Fig. 7a (8a) display A^{int} for d/a = 3 (6). The interaction between scattering by the sphere and a single reflection by the plane induces variation as a function of θ_o. The dependence on θ_o at fixed frequency, as well as on αa for fixed θ_o is sensitive to the distance d. It is evident that these processes have the largest relative contribution to the scattered amplitude. The processes of Figs. 5a-5c contribute A^{PSP}, presented on Figs. 7b and 8b, and the total amplitude is shown in Figs. 7c and 8c.

The main advantage of our analysis is that it provides a fast and computationally inexpensive approximate solution to the problem. By using formal expressions such as equations (9)-(12), the need for expensive and tedious numerical momentum-space integrations is eliminated. However, one should bear in mind the approximations that were made. First, inclusion of the Shear-Shear process of Fig. 5d is needed. This can easily be done, using the appropriate solution of the spherical cavity (with incident shear wave) problem. The only difficulty is to make sure that all signs and phase factors are consistent with the evaluation of the scattering problem with longitudinal incident wave. (Note that by using reciprocity, all scattering amplitudes used in this report are derived from the latter problem only). We shall include the contribution of Fig. 5d in our next report, and do not expect it to alter our results in a profound manner.

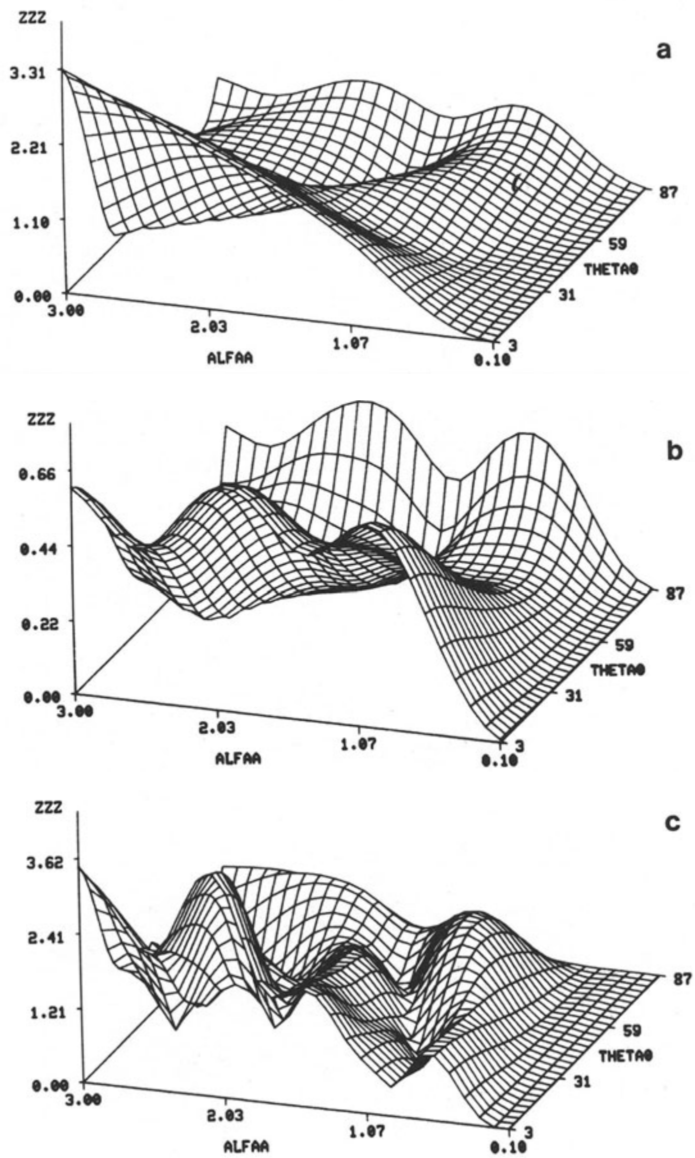

Fig. 7 The amplitudes (a) A^{int}, (b) A^{PSP} and (c) A^{Scatt} for d/a=3, as functions of αa and θ_o.

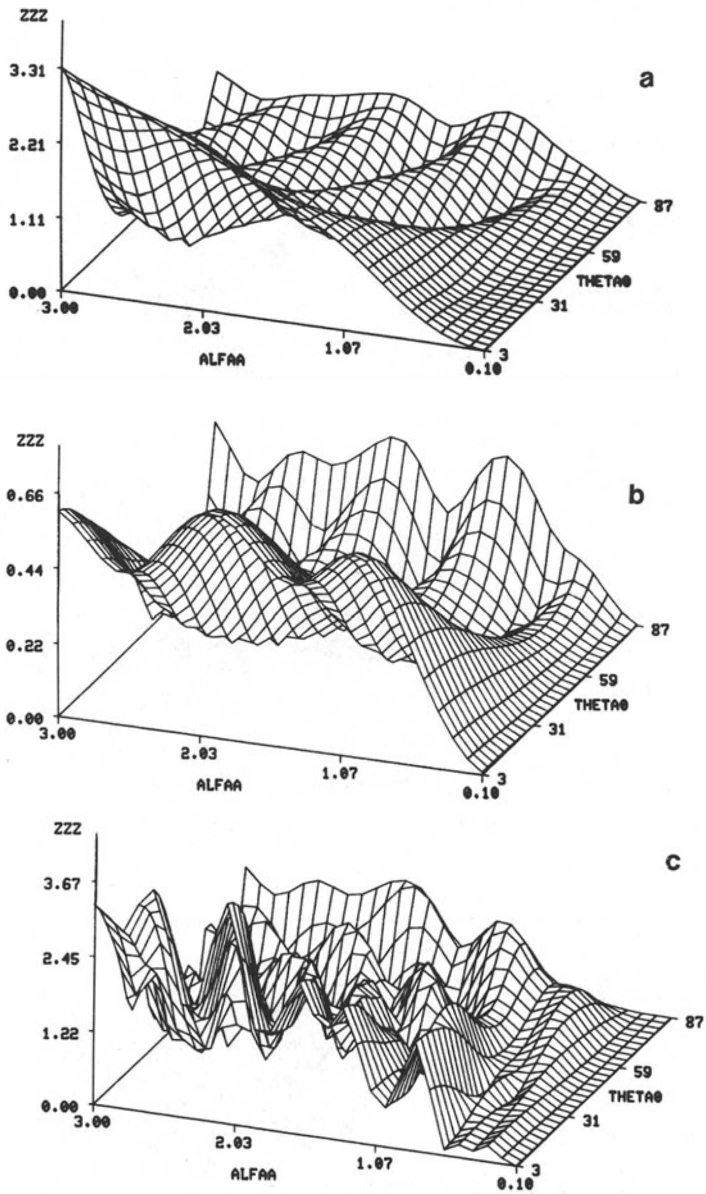

Fig. 8 The amplitudes (a) A^{int}, (b) A^{PSP} and (c) A^{Scatt} for $d/a=6$, as functions of αa and θ_o.

As to the processes that were neglected which involve multiple scattering by the sphere - we have shown in the past that their contribution is negligible for d/a $\gtrsim$ 2.

We also plan to extend our work to liquid-metal interfaces, curved surfaces and non spherical defects. Also, we plan to calculate the time response to a δ-function impulse, to check the separability, on the basis of arrival times, of the process of Figs. 2-5.

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